Cyclic Homology of an Isolated Hypersurface Singularity

by R. Buchweitz and D. van Straten

Introduction

For any A-algebra B there exist groups $HC_*(B/A)$, called the Cyclic Homology groups of B over A (see [Ca], [L&Q]). These groups can be constructed with the help of the *exterior derivation* operator on the Hochschild complex. For example, when X is a smooth affine variety in $\mathbb{C}^N$ and B is the ring of X, then one has:

$$HC_i(X) = \Omega^i_X / d\Omega^{i-1}_X \oplus H^{i-2}(X,\mathbb{C}) \oplus H^{i-4}(X,\mathbb{C}) \oplus \ldots .$$

where $HC_*(X) := HC_*(B/\mathbb{C})$. We see that Cyclic Homology contains apart from purely topological contributions also some analytic information. Because of the relation with algebraic K-theory ([F&T]), characteristic classes ([K]) and thus algebraic cycles, this theory aroused considerable interest and gave hope to get more computable invariants.

For this reason it is of interest to compute the Cyclic Homology for the simplest non - trivial case: the case of the local ring of an isolated hypersurface singularity. In [F&T] Cyclic Homology (or rather *additive K - theory*) for the more general case of isolated complete intersection singularities is computed in terms of Crystalline Cohomology. In this note we will be more explicite in our answer. In [Bry] one can find computions of the Cyclic Homology of some interesting non - commutative rings.

In ordinary de Rham theory the exterior derivation operator **d** can be considered as the basic link between Algebra and Topology. In our computation of Cyclic Homology we use transcendental - topological arguments first introduced by Brieskorn ([Bri]) in the study of the local Gauß - Manin connection. We do not know of any purely algebraic proofs of these results.

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Notations and Conventions

In the rest of this note we use the following notations and conventions. We put:

$$\begin{aligned} B &= k\langle x_0, x_1, \ldots, x_n\rangle \ , \\ A &= k\langle t\rangle \ , \\ f &\in B \ , \\ B_0 &= B/(f) \ , \\ A_0 &= k \ . \end{aligned}$$

B is considered as an A-module by letting t act as multiplication by the element f, the equation of our hypersurface. The brackets $\langle\ \rangle$ can mean $[\![\ ]\!]$, { } , $[\ ]_{(0)}$, etc. and sometimes even [ ]. Tensor products should be read according to this context. When we refer to the whole set of variables x_j we sometimes use a single symbol $\mathbf{x}$. The symbol X will stand for the variety defined by the function f.

In § 1 we compute relative and absolute Hochschild Homology (i.e. of B over A and of B_0 over A_0). For this, k can be any field. In § 2 we compute relative and absolute Cyclic Homology and here we have to assume $k = \mathbb{C}$, but undoubtedly the statements are true over any reasonable field when they make sense.

We remark that for *isolated* hypersurface singularities it does not really matter which local ring we take, but for *non - isolated* singularities it probably does matter.

§ 1. Hochschild Homology

The Hochschild homology $H_*(B/A)$ of B over A is defined as the homology of the Hochschild complex $(\mathcal{B}^\bullet, b)$ (see [C&E]) :

$$\mathcal{B}^n = B \otimes_A B \otimes_A \ldots \otimes_A B \quad ; \quad (n+1)\text{- times} \tag{1.1}$$

$$b(a_0 \otimes a_1 \otimes \ldots \otimes a_n) = (-1)^n \, a_n . a_0 \otimes a_1 \otimes \ldots \otimes a_{n-1} \; + \\ + \sum (-1)^{i+1} \, a_0 \otimes \ldots \otimes a_i . a_{i+1} \otimes \ldots \otimes a_n \, .$$

Of course, it is quite impossible to compute with this definition, but when B is A-flat then $H_*(B/A) = \mathrm{Tor}_*^U(B,B)$; $U = B \otimes_A B$, so we really only need a flat resolution of B as a U - module. With A and B as in the introduction we have:

$$B \otimes_A B = P/F \text{ with } P := B \otimes_k B \approx k\langle \mathbf{y}, \mathbf{z} \rangle \tag{1.2}$$

and $\qquad F = f \otimes 1 - 1 \otimes f = f(\mathbf{y}) - f(\mathbf{z})$.

Here $\mathbf{y}$ and $\mathbf{z}$ are $(n+1)$-tuples of variables.
To get a resolution of B over U one can first resolve B over P by a Koszul complex. As U is a hypersurface quotient of P one can obtain a resolution over U by the method of [Ei]. We prefer an essentially equivalent method: we first replace U by a differential graded ring $P[\mathbf{e}]$, with $\mathbf{e}$ a new element in degree 1 and a differential ∂ such that $\partial\, \mathbf{e} = F$. The kernel of the obvious map $P[\mathbf{e}] \longrightarrow B$ is $(\mathbf{e}, \mathbf{y}-\mathbf{z})P[\mathbf{e}]$, so it is generated by a regular sequence. Hence, B is resolved over $P[\mathbf{e}]$ by a Koszul complex:

$$P[\mathbf{e}] \otimes \textstyle\bigwedge^\bullet d\mathbf{x} \otimes \Gamma_\bullet \, d\mathbf{e} \tag{1.3}$$

This complex has a bigrading $(|e|,|d|)$ by counting the number of e's and the number of d's in an expression. The complex carries two differentials. In the first place there is the Koszul differential δ of degree $(0,-1)$, defined by the relation $\delta(d\mathbf{x}) = \mathbf{y} - \mathbf{z}$. (We do not write indices for $\mathbf{x}$, $\mathbf{y}$ or $\mathbf{z}$). Further, there is a differential induced by ∂ of degree $(-1,0)$. When we write

$$f(\mathbf{y}) - f(\mathbf{z}) = \sum J_i \, . \, (y_i - z_i) \tag{1.4}$$

for some choice of the J_i , then one can take:

$$\partial(de) = \sum J_i . \, dx_i \, . \tag{1.5}$$

To get the Hochschild homology we tensor this complex with B, which means that we put everywhere **y** and **z** equal to **x**. Hence, the Koszul differential vanishes identically and the J_i are replaced by the partial derivatives of f with respect to the coordinates x_i . Because $\Gamma_p(de)$ is just a vector space of dimension 1, we may suppress this from our notation. The conclusion now is:

Proposition (1.1) : The Hochschild homology of B over A is isomorphic to the cohomology of the total complex $Tot(L_{..})$, where $L_{..}$ is the double complex with terms

$$L_{pq} = \Omega^{p-q} \; ; \; \Omega^r := B \otimes \bigwedge^r (dx) = \Omega^r_{B/k} \tag{1.6}$$

and with $df\wedge : L_{pq} \longrightarrow L_{p-1\,q}$ as horizontal differential and zero as vertical differential.

Note that the fact that one differential is zero gives rise to a *direct sum decomposition* in the Hochschild homology. So far this holds for every f.

Lemma (1.2) : For an *isolated hypersurface singularity* f the complex $(\Omega^\bullet, df\wedge)$ only has homology in degree n+1.

proof : The partial derivatives of f form a regular sequence and the above complex can be identified with a Koszul - complex. In fact, one can take this as a definition of an isolated hypersurface singularity. ⊠

This unique non-zero cohomology group $\Omega^{n+1}/df\wedge\Omega^n$ is a finite dimensional vector space over k. Its dimension is usually denoted by μ and is called the *Milnor number* of the singularity (see [Lo]).

Put $\Omega^p_f := \Omega^p/df\wedge\Omega^{p-1} \approx \Omega^p_{B/A}$. These are the so-called *relative differential forms* of the map f. When we use this notation, then the above proposition and lemma together give the following result:

Theorem (1.3) : The Hochschild homology of an isolated hypersurface singularity is given by:

$$H_i(B/A) = \Omega^i_f \quad ; \; i = 0,1,\dots,n+1 \; ,$$

$$\left.\begin{array}{ll} H_{n+1+2k}(B/A) & = \Omega^{n+1}_f \\ H_{n+1+2k+1}(B/A) & = 0 \end{array}\right\} \quad k = 0,1,\dots \; .$$

To compute the absolute Hochschild homology $H_*(B_0/A_0)$ one can use the following easy to prove general fact.

Lemma (1.4) : Let
$$\begin{array}{ccc} B & \longrightarrow & B_0 \\ \uparrow & & \uparrow \\ A & \longrightarrow & A_0 \end{array}$$
be a base change diagram of rings, with B an A-flat module. Then there is a spectral sequence

$$E^2_{pq} = \mathrm{Tor}^{A_0}_p(H_q(B/A), B_0) \Rightarrow H_{p+q}(B_0/A_0) .$$

Corollary (1.5) : For the rings under consideraton we have a long exact sequence:

$$\ldots \longrightarrow H_{k+1}(B_0/A_0) \longrightarrow H_k(B/A) \xrightarrow{f.} H_k(B/A) \longrightarrow H_k(B_0/A_0) \longrightarrow \ldots$$

From this exact sequence we see that to compute absolute Hochschild homology, we need to know how multiplication by the element f acts on the relative Hochschild homology. For this we use the following simple lemma:

Lemma (1.6) : depth $(\Omega^p_f) \geq n+1-p$.

proof: By the de Rham lemma, the sequences

$$0 \longrightarrow \Omega^{p-1}_f \xrightarrow{df\wedge} \Omega^p \longrightarrow \Omega^p_f \longrightarrow 0$$

are exact for $p = 1, 2, \ldots, n+1$. Now use induction starting with depth(B)=n+1 . ⊠

So in particular, we see that all the Ω^p_f , $p = 0, 1, \ldots, n$, are torsion free, hence multiplication by f is injective. Only on Ω^{n+1}_f multiplication by f has a kernel and a cokernel, which we denote by K and T respectively. Remark that K and T have the same dimension. This dimension is usuallly called the *Tjurina number* of the singular point (see [Lo]). This also is the dimension of the space of *first order deformations* of the germ X. As a corollory of the above results we get:

Theorem (1.7) **:** The absolute Hochschild homology $H_*(B_0/A_0)$ of an isolated hypersurface singularity X is given by:

$$H_i(B_0/A_0) = \Omega^i_X \quad ; \quad i = 0, 1, \ldots, n+1$$

$$\left.\begin{array}{ll} H_{n+1+2k}(B_0/A_0) & = T \\ H_{n+1+2k+1}(B_0/A_0) & = K \end{array}\right\} \quad k = 0, 1, \ldots$$

Remark (1.8) **:** For a general complete intersection defined by functions $f_1, f_2, \ldots, f_k \in B = k\langle x_0, x_1, \ldots, x_{n+k}\rangle$, $A = k\langle t_1, t_2, \ldots, t_k\rangle$ one can still define a complex like $L_{..}$ to compute the Hochschild homology. In that case one finds that:

$H_k(B/A) = \Omega^k_{B/A}$, for $k \leq c$: = codimension of the singular locus.

But the cohomology for $k \geq c+1$ seems to lack any interesting interpretation.

Remark (1.9) **:** The fact that $L_{..}$ appears as a direct sum of complexes gives a direct sum decomposition (trivial in our case) for Hochschild homology. This is a very general phenomenon in characteristic 0. In fact, a theorem of Quillen [Q] says:

$$H_n(B/A) = \bigoplus_{p+q=n} H_p(\Lambda^q \mathbf{L}_{B/A})$$

where $\mathbf{L}_{B/A}$ is the *cotangent complex* of B over A.

§ 2. Cyclic Homology

Cyclic homology is defined as the homology of a certain double complex (see [L&Q]). We call this double complex ($\mathfrak{C}_{..}$; b, R). It has the following shape:

$$\begin{array}{ccccc} \vdots & & \vdots & & \vdots \\ \downarrow & & \downarrow & & \downarrow \\ B^{\otimes 3} & \xleftarrow{R} & B^{\otimes 2} & \xleftarrow{R} & B \\ b\downarrow & & b\downarrow & & \\ B^{\otimes 2} & \xleftarrow{R} & B & & \\ b\downarrow & & & & \\ B & & & & \end{array} \qquad (2.1)$$

The verticals of this complex are formed Hochschild complexes (1.1) of § 1. R is a certain A-linear map and is called Connes' operator. (Usually this operator is denoted by B, but for obvious reasons we choose another symbol). In as far Hochschild homology can be seen as a generalization of Kähler differentials, this operator R can be seen as a generalization of *exterior derivative*.

Now we want to compute the cyclic homology $HC_*(B/A)$ for the rings A and B we have in mind. As we already know the Hochschild homology (Theorem (1.3)) it is tempting to try to use the spectral sequence:

$$E^1_{pq} = H_{q-p}(B/A) \Rightarrow HC_{p+q}(B/A) \qquad (2.2)$$

which arizes from (2.1). The problem with this approach is that it is not totally clear what the higher differentials are. However, it is convenient to replace the columns of the above complex by the complex $\mathrm{Tot}(L_{..})$ of §1, (1.6) which is quasi-isomorphic to it. It turns out that the operator R can be lifted to the $L_{..}$ - level, where it can be identified with ordinary exterior differentiation of forms. Thus one gets:

Lemma (2.1) : Let $C_{...}$ be the triple complex with

$$C_{pqr} = \Omega^{p-q-r} \quad \text{as terms}$$

and

$$df\wedge : C_{pqr} \longrightarrow C_{pq\,r-1} \;,$$
$$d \;: C_{pqr} \longrightarrow C_{p\,q-1\,r}$$

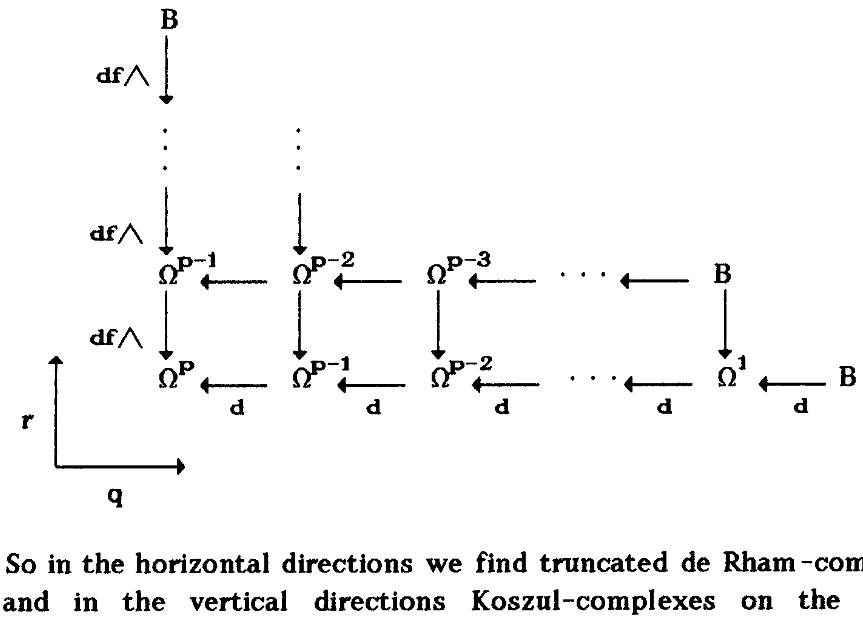
as differentials . (The third differential is the zero map.).
Then Tot($C_{...}$) and Tot($\mathcal{C}_{..}$) are quasi-isomorphic.

So we can use the complex $C_{...}$ to compute $HC_*(B/A)$. As one of the differentials of the complex $C_{...}$ is the zero map, we get a direct sum decomposition for cyclic homology. Let us analyze the layers $C_{p..}$ for various values of p . Each such layer contributes a summand in $HC_*(B/A)$:

$$HC_m(B/A) = \bigoplus_{p+q=m} H_q(\,\mathrm{Tot}(C_{p..};\, df\wedge, d))\;. \tag{2.3}$$

The layers $C_{p..}$ have the following form:

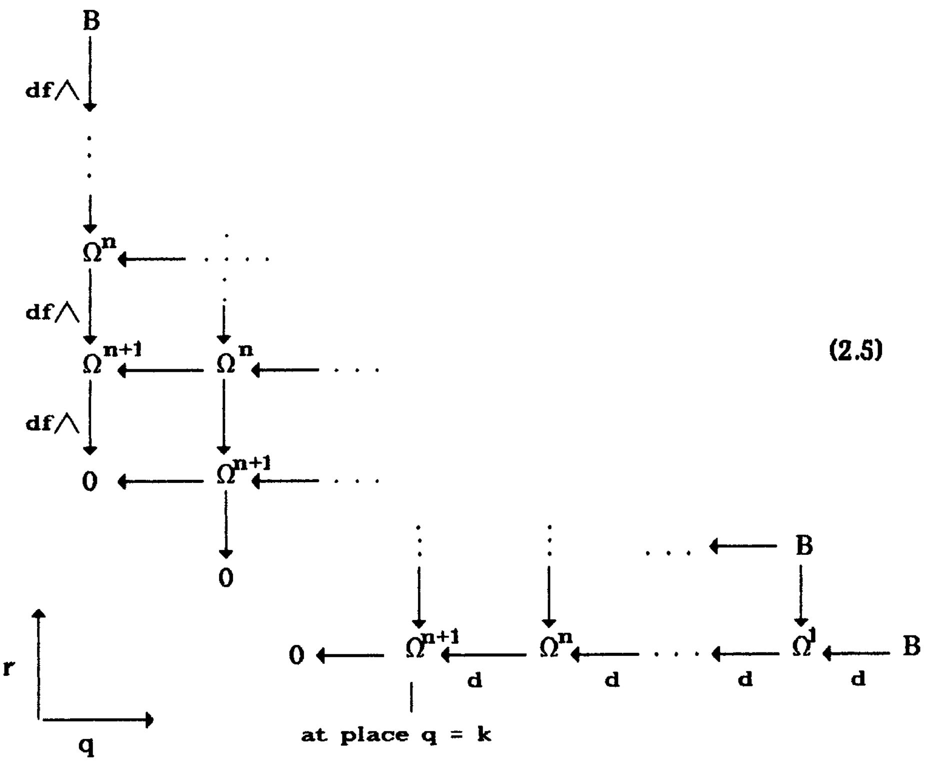
Case A , triangular shape : $p \le n + 1$



(2.4)

So in the horizontal directions we find truncated de Rham-complexes and in the vertical directions Koszul-complexes on the partial derivatives of the function f.

Case B, trapezium shape : $p = n + 1 + k \; ; \; k \geq 0$.



(2.5)

So in this case we also get some complete de Rham complexes.

We now compute the cohomology of each layer, using the spectral sequence of the colomn filtration. These columns do not carry much cohomology, by lemma (1.2). The result is:

Case A : $E^1_{q\,r} = \Omega_f^{p-q}$ for $r = 0$ and $q = 0, 1, \ldots, p$.
$\qquad\quad = 0$ otherwise.

Case B : $E^1_{q\,r} = \Omega_f^{n+1}$ if $q + r = k$
$\qquad\quad = \Omega_f^{p-q}$ for $r = 0$
$\qquad\quad = 0$ otherwise.

To compute further in the spectral sequence, we need to know at least the cohomology of the *relative de Rham complex* $(\Omega^{\cdot}_f \; ; \; d)$:

$$0 \longrightarrow B \longrightarrow \Omega^1_f \longrightarrow \cdots \longrightarrow \Omega^n_f \longrightarrow \Omega^{n+1}_f \longrightarrow 0 \qquad (2.6)$$

Fortunately, the cohomology of this complex is known, and by now classic. We quote the result.

Theorem (Brieskorn - Sebastiani) : The cohomology of the relative de Rham complex (2.6) is as follows:

$H^i(\Omega_f) = 0$ for all $i \neq 0, n$,

$H^0(\Omega_f)$ is a free A-module of rank 1 ,

$H^n(\Omega_f)$ is a free A-module of rank $\mu = \dim(\Omega_f^{n+1})$.

For a proof of this result, see [Bri] or [Lo]. Especially the fact that $H^n(\Omega_f)$ is a free module seems to be rather deep.

We remark that although the statement of the above result is purely algebraic we do not know of any proof which does not use at some point transcendental-topological methods. So at this point we should take $k = \mathbb{C}$ and $\langle\ \rangle = \{\ \}$ or $[\![\ ]\!]$.

Besides the group H :=

$$H^n(\Omega_f) = \{ \omega \in \Omega^n |\ \exists\eta \text{ with } d\omega = df\wedge\eta \} / (df\wedge\Omega^{n-1} + d\Omega^{n-1})$$

Brieskorn introduced further groups H' and H":

$$H' := \Omega^n / (df\wedge\Omega^{n-1} + d\Omega^{n-1}) ,$$
$$H'' := \Omega^{n+1} / df\wedge d\Omega^{n-1} . \tag{2.7}$$

These groups are also free A-modules of rank μ. H, H' and H" are usually called the *Brieskorn modules* of the singularity. There are also A-linear inclusion maps:

$$H \hookrightarrow H' \quad ; \quad \omega \longmapsto \omega \tag{2.8}$$
$$H' \hookrightarrow H'' \quad ; \quad \omega \longmapsto df\wedge\omega$$

and (only $\mathbb{C}$-linear) operators ∂_t:

$$\partial_t : H \xrightarrow{\approx} H' \quad ; \quad \omega \longmapsto \eta \tag{2.9}$$
$$\partial_t : H' \xrightarrow{\approx} H'' \quad ; \quad \omega \longmapsto d\omega .$$

So the groups H, H', H" are related by a chain of inclusions (2.8). The operator ∂_t is called the *local Gauß-Manin connection* and induces $\mathbb{C}$-vector space isomorphisms between these groups. Further, it is easy to see that one has natural identifications:

$$H'/H \xrightarrow[\partial_t]{\approx} H''/H' \approx \Omega_f^{n+1} . \tag{2.10}$$

The group H can be used to describe n-cohomology classes in the f fibres (dual to the vanishing cycles). The operator ∂_t then describes the differentiation of period integrals. It should be kept in mind that ∂_t behaves formally as differentiation of forms with respect to f = t. For more details we refer to [Ph].

After this short intermezzo, we can go on with our computation of the cohomology in a layer $C_{p..}$. In *Case A* , there is no problem: the spectral sequence degenerates at E^2. The result is:

Case A : $p \leq n+1$.

$$E^2_{qr} = E^{\infty}_{qr} = A \qquad \text{for } q=p \text{ and } r=0$$

$$= \Omega_f^p / d\Omega_f^{p-1} \qquad q=r=0$$

$$= 0 \qquad \text{otherwise} .$$

Case B : p = n+1+k.

$$E^2_{qr} = \Omega_f^{n+1} \qquad \text{for } q+r=k \text{ and } r \geq 1$$

$$= H \qquad q=k+1 \text{ and } r=0$$

$$= A \qquad q=p \text{ and } r=0$$

$$= 0 \qquad \text{otherwise} .$$

In this last case there is a non-zero d_2 in the spectral sequence. It is a map $d_2 : H \longrightarrow \Omega_f^{n+1}$. Running through the definitions, we see that it is represented by the following operation:

$$d_2 : H \longrightarrow \Omega_f^{n+1} \; ; \; \omega \text{ with } d\omega = df \wedge \eta \longmapsto d\eta . \tag{2.11}$$

As every (n+1)-form is the d of some n-form, we see that the map d_2 is *surjective*. The kernel of d_2 can be identified with the set of elements ω in H for which $\partial_t\,\omega$ is still in H (instead of in the bigger space H'). It is sensible to denote this kernel by ∂_t^{-1} H. It is again a free A-module of rank μ. So the effect of the d_2 differential is to kill off a copy of Ω_f^{n+1} and to replace H by ∂_t^{-1} H. The same pattern is repeated for the higher differentials of the spectral sequence. We get the following result:

Case B : $p = n+1+k$.

$$E^{k+2}_{qr} = E^{\infty}_{qr} = \partial_t^{-k} H \quad \text{for} \quad q = k+1 \text{ and } r = 0$$
$$= A \qquad q = p \text{ and } r = 0$$
$$= 0 \qquad \text{otherwise} .$$

This completes the computation of the cohomology of the layers $C_{p\cdot\cdot}$. The decomposition formula (2.3) now tells us that in fact we have computed the cyclic homology HC_* of B over A. Putting things together we find the following theorem.

Theorem (2.2) : The relative Cyclic Homology $HC_*(B/A)$ of an isolated hypersurface singularity is given by:

$$\left.\begin{array}{ll} HC_i(B/A) = \Omega_f^i / d\Omega_f^{i-1} & (i \le n) \\ HC_{n+1+2k}(B/A) = 0 & \\ HC_{n+1+2k+1}(B/A) = \partial_t^{-k} H & \end{array}\right\} \oplus \text{ A in even degrees.}$$

We thus find in degrees $\le$ n the same groups as if B were smooth over A. It is only in the *stable range* (degree $\ge$ n+1) that the presence of the isolated singular point is detected by cyclic homology. As an A-module HC_{n+1+*} (B/A) alternates between 0 and a free module of rank μ, the Milnor number of the singularity (plus of course A in even degrees).

From (2.1) one can derive an exact sequence relating Cyclic and Hochschild homology (see [L&Q]):

$$\cdots \longrightarrow H_k(B/A) \xrightarrow{I} HC_k(B/A) \xrightarrow{S} HC_{k-2}(B/A) \xrightarrow{R} H_{k-1}(B/A) \longrightarrow \cdots \tag{2.12}$$

It is instructive to combine the *(1.3)* and *(2.2)* with (2.12). The relevant piece in the stable range reads as follows:

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$$\begin{array}{ccccccccc} \cdots \longrightarrow & H_{n+1+2k+1} & \longrightarrow & HC_{n+1+2k+1} & \xrightarrow{S} & HC_{n+1+2k-1} & \longrightarrow & H_{n+1+2k} & \longrightarrow \cdots \\ & \| & & \| & & \| & & \| & \\ & 0 & \longrightarrow & \partial_t^{-k} H & \longrightarrow & \partial_t^{-k+1} H & \longrightarrow & \Omega_f^{n+1} & \longrightarrow 0 \end{array}$$

From this we see that the suspension operators S here correspond to the canonical inclusions of the system $\ldots \subset \partial_t^{-2} H \subset \partial_t^{-1} H \subset H$. It is known that $\bigcap \partial_t^{-k} H = \{0\}$ (see [Ph]), so making S invertible would kill all interesting information.

Finally, we can compute the absolute cyclic homology by looking at the action of multiplication by f on the relative groups. This is completely analoguous to the arguments used to get theorem *(1.7)* from *(1.3)*. The only crucial point is that the Brieskorn module H is A-free. The result is:

Theorem (2.3) : The absolute Cyclic homology $HC_*(B_0/A_0)$ of an isolated hypersurface singulartity is given by:

$$\left.\begin{array}{lcl} HC_i(B_0/A_0) & = & \Omega_X^i / d\Omega_X^{i-1} \quad (i \le n) \\ HC_{n+1+2k}(B_0/A_0) & = & 0 \\ HC_{n+1+2k+1}(B_0/A_0) & = & \partial_t^{-k} H / t.\partial_t^{-k} H \ (\approx \mathbb{C}^{\mu}) \end{array}\right\} \oplus \mathbb{C} \text{ in even degrees}$$

This copy of $\mathbb{C}$ in the even degrees of course can be identified naturally with $HC_*(\mathbb{C}/\mathbb{C})$. The factors $\partial_t^{-k} H/t.\partial_t^{-k} H$ are, just as Ω_f^{n+1} is, vector spaces of dimension μ. However there is no natural way to identify these spaces with each other. It is easy to verify that one has the following isomorphisms:

$$\partial_t^k \;:\; \partial_t^{-k} H/t.\partial_t^{-k} H \xrightarrow{\approx} H/(t+k.\partial_t^{-1})H \tag{2.14}$$

Remark (2.4) : In the case that the hypersurface f is *quasi-homogeneous* the situation is much simpler and one in fact can avoid reference to the theorem of Brieskorn-Sebastiani. The reason is that the operator ∂_t^{-1} can be described in simple terms: if $\omega \in H$ is an n-form of quasi-homogeneous weight α, then $\partial_t^{-1}\,\omega = (t/\alpha).\omega$. (This can be seen by applying the Lie-derivative with respect to the Euler vector field to the defining relation $d\,\partial_t^{-1}\,\omega = df \wedge \omega$.) A further simplification arises in the quasi-homogeneous case: by the $\mathbb{C}^*$-action every local result is actually globally true. So theorems *(2.2)* and *(2.3)* are true for $\langle\, ,\rangle = [\; ,\;]$ in the quasi-homogeneous case.

Remark (2.5) : In the theory of the Gauß-Manin connection associated to a function f one studies the so-called *Gauß-Manin system*. This is the $\mathcal{D}$-module which is the cohomology of the complex $(\,\Omega^{\cdot}[D],\, \mathbf{d}\,)$ on which there are operators t and ∂_t . (see [Ph], p.159). This complex bears some relation to the layers $C_{p\cdot\cdot}$ (2.4) and (2.5). Both complexes are truncations of the bi-infinite double complex based on $\Omega^{\cdot}$ and the operators d and $df\wedge$. It would be interesting to find a general relation between $\mathcal{D}$-modules and Cyclic Homology.

R. Buchweitz

Department of Mathematics
University of Toronto
100 St. George street
M5S 1A4 Ontario
Canada.

D. van Straten

Mathematisch Instituut
Rijksuniversiteit Utrecht
Budapestlaan 6
3508 TA Utrecht
The Netherlands.

References

[Bry] : Brylinski, J.-L.; *Some examples of Hochschild and Cyclic homology* . In LNM 1271 (Algebraic Groups Utrecht '86, ed. Cohen c.s), Springer '87

[Bri] : Brieskorn, E. ; *Die Monodromie der Isolierte Singularitäten von Hyperflächen,* Manus. Math. 2, (1970), 103-161.

[Ca] : Cartier, P. ; *Homologie cyclique : Rapport sur des travaux recents de Connes, Karoubi, Loday, Quillen,...,* Sem. Bourbaki, exp. 621, (1984)

[C&E] : Cartan, H. & Eilenberg, S. ; *Homological Algebra,* Ann. of Math. Studies, vol.19, (Princeton University Press, 1956).

[Ei] : Eisenbud, D. ; *Homological algebra on Complete Intersections with an application to Group Representations,* Trans. AMS 260, 131-163.

[F&T] : Feigin, B.L. & Tsygan, B.L. ; *Additive K - theory and Chrystalline Cohomology,* Func. Anal. Appl. Vol 19, No.2 (1985), 124-132.

[K] : Karoubi, M.; *Homologie cyclique et K- theorie algebrique, I and II, C. R. Acad. Paris,* serie I, vol 297 (1983), 447-450 & *513-516.*

[Lo] : Looijenga, E. ; *Isolated Singular Points on Complete Intersections.* London Math. Soc. Lect. Notes 77, (Cambridge University Press, 1984).

[L&Q] : Loday, J.-L. & Quillen, D. ; *Cyclic homology and the Lie Algebra homology of Matrices,* Comm. Math. Helv. 59 (1984), 565-591.

[Ph] : Pham, F. ; *Singularites des systemes differentiels de Gauss - Manin* Progress in Math. vol2, (Birkhauser, 1979).

[Q] : Quillen, D. ; *On the (Co-) Homology of commutative rings,* In: Applications of Categorical Algebra, Proc. Symp. Pure. Math. Vol. XVII, A.M.S. 1970.